SRS Student Symposium at USU

PRE Expanders: Handling More Than Just Pressure

A derivation of planetary rotary expander (PRE) isentropic efficiency with varying machine size and speed.

By Joseph James

Mechanical Engineering MS Graduate Student

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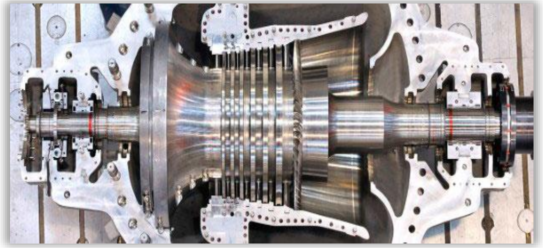
Expanders: Types and Limitations

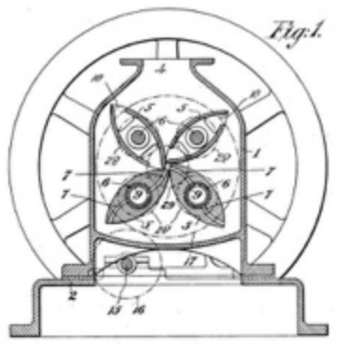


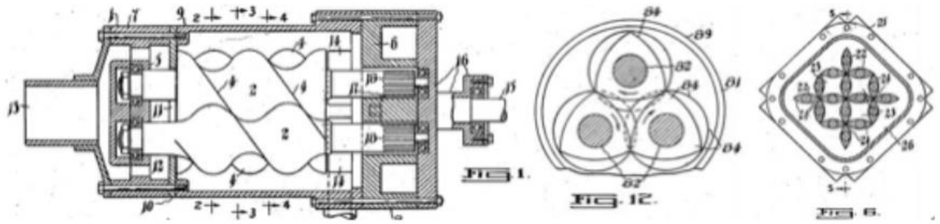
	Turbine	Twin Screw	Reciprocating	PRE
Economical	✗	✓	✓	✓
High Pressures	✓	✗	✓	✓
High Flowrates	✓	✗	✓	✓
Mix/Variant Flows	✗	✓	✗	✓
Oil Free	✓	✓	✗	✓
Self Cleaning	✗	✗	✗	✓

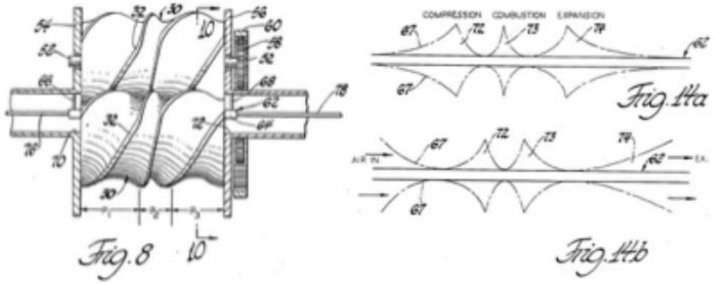
All required for use in industrial applications.

1 http://helidynepower.com/applications/
2 https://commons.wikimedia.org/wiki/File:Internal_combustion_engine_pistons_of_partial_cross-sectional_view.jpg
2 https://dir.indiamart.com/ahmedabad/screw-compressors.html
3 https://commons.wikimedia.org/wiki/File:Internal_combustion_engine_pistons_of_partial_cross-sectional_view.jpg

Not New, Just Undeveloped: The PRE History







1902
Thomas S. Colbourne
Patent for a planetary rotary engine

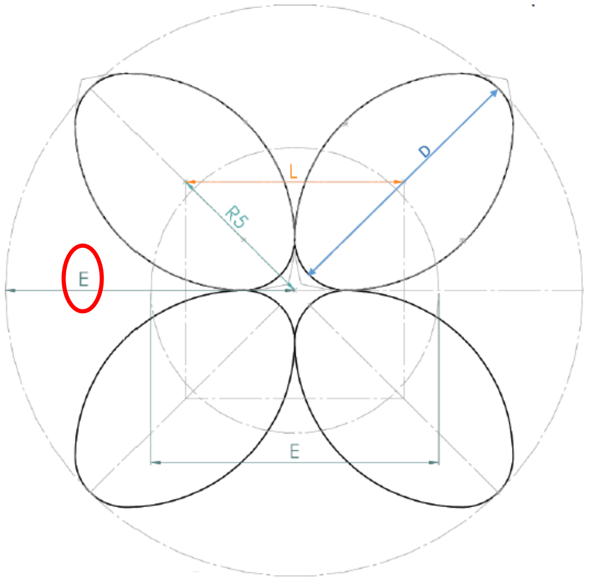
1946
Rudolph D. Delamere
Patent helical twist on the rotors, 3 rotor configuration, matrix design

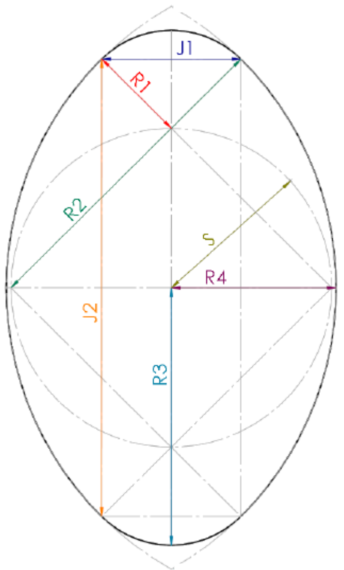
1988
Constantinos A. Koromilas
Patent for variable pitch, multi-twist configuration

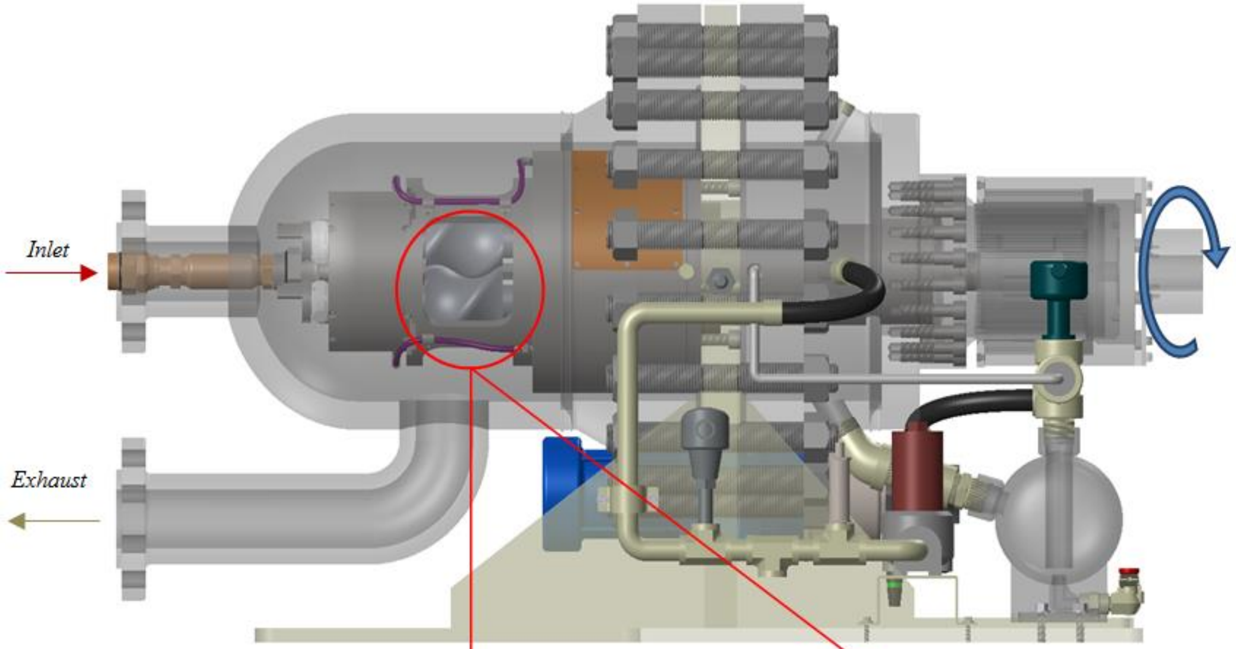
1900 1950 2000

2014
Helidyne
Patent for drive assembly.
First documented build of the planetary rotary expander.

How It Works

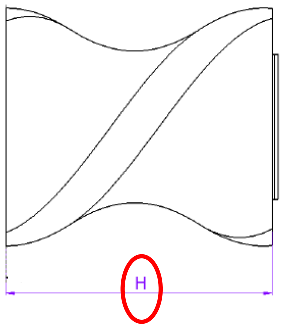






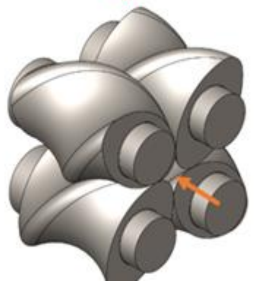
Primary Rotor Parameters

E	=	Machine Radius
H	=	Rotor Height
ƒ	=	Rotational Speed



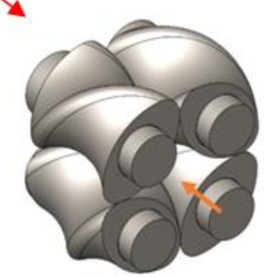
Rotors at zero torque position

(cycle starting position)



Rotors at full torque position

(half cycle or quarter turn)

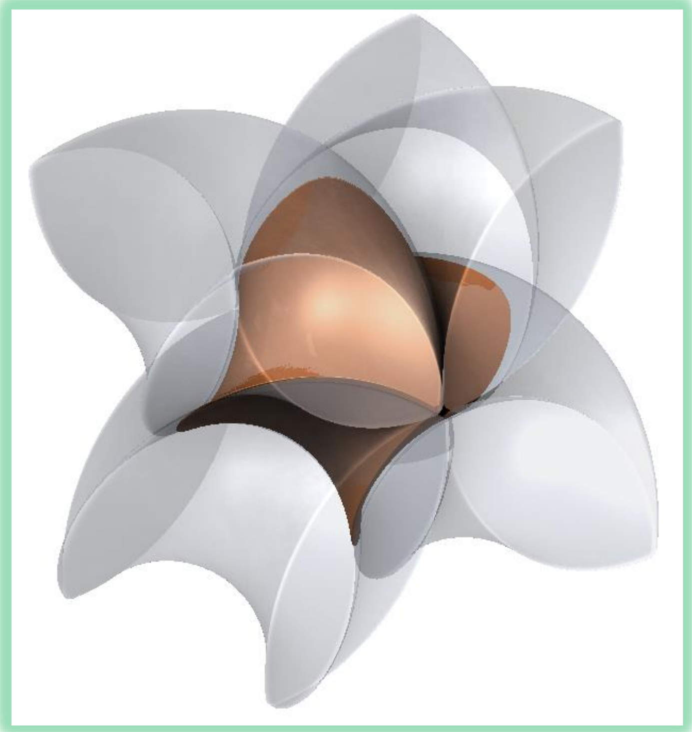


Rotor parameters determine machine efficiency. Optimization is needed to maximize efficiency.

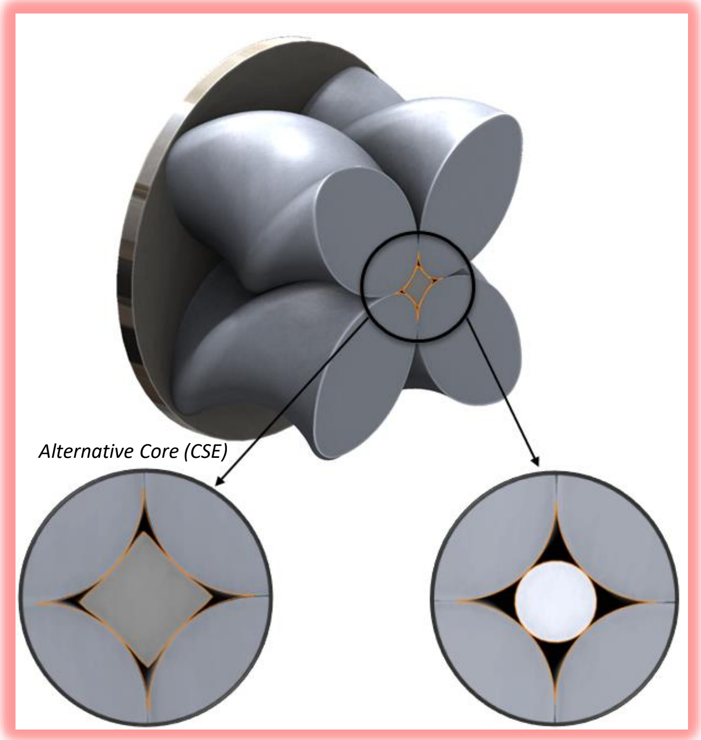
Isentropic Efficiency Model

$$E_{system} = \text{Isentropic Efficiency}$$

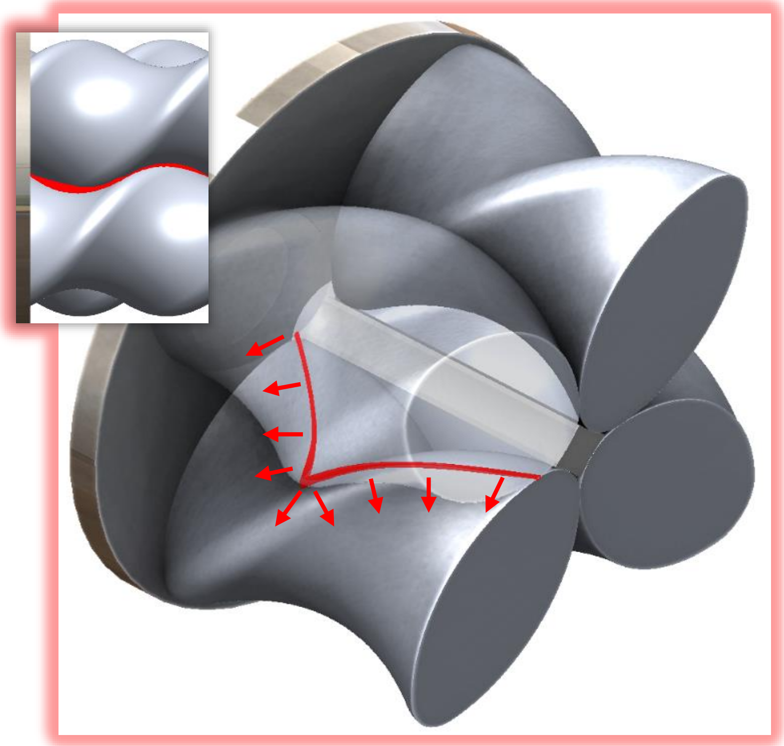
$$E_{system} = \frac{\dot{W}_{shaft}}{\dot{W}_{system}} = \frac{\dot{V}_{cav.net}\,(P_2 - P_3)}{\dot{m}_{tot}\left(h_1 - h_{3,Isentropic}\right)}$$



Cavity Volume



Core Leakage



Gap Leakage

Isentropic Efficiency Mathematical Derivation

$$\mathbf{E}_{\mathbf{system}} = \frac{\dot{w}_{shaft}}{\dot{w}_{system}} = \frac{\dot{V}_{cav.net}\,(P_2 - P_3)}{\dot{m}_{tot}\,(h_1 - h_{3,Isentropic})}$$

Results of Geometric Study

$$\dot{V}_{cav.net} = Hf\left[\left(2-\frac{\pi}{2}\right)(E-R_1)^2 - \pi R_1^2\left(1-\sqrt{2}\right)^2\right]$$

$$\dot{m}_{tot} = \dot{m}_{leakage} + \dot{m}_{cavity}$$

$$\dot{m}_{cavity} = \rho_2 \dot{V}_{cav.net}$$

$$\dot{m}_{leakage} = \rho_t\sqrt{\gamma_t R_g T_t}\left(\underbrace{2G\sqrt{\left[\pi T_{coil}\left(\frac{2E-R_1\sqrt{2}}{2}\right)\right]^2 + H^2}}_{\textit{Gap Leak Area}} + \underbrace{R_1^2\left[4-\pi-\pi\left(1-\sqrt{2}\right)^2\right]}_{\textit{Cylindrical Core Leak Area}}\right)$$

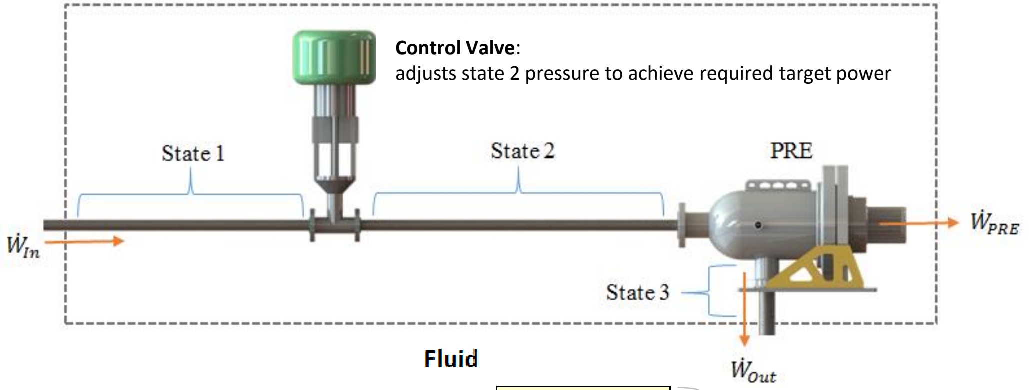
Cannot be solved analytically. Iterative techniques must be employed to solve throat conditions and cavity pressure.

$$\mathbf{E}_{\mathbf{system}} = \frac{Hf\left[\left(2-\frac{\pi}{2}\right)(E-R_1)^2 - \pi R_1^2\left(1-\sqrt{2}\right)^2\right](P_2 - P_3)}{\left[\rho_2 Hf\left[\left(2-\frac{\pi}{2}\right)(E-R_1)^2 - \pi R_1^2\left(1-\sqrt{2}\right)^2\right] + \rho_t\sqrt{\gamma_t R_g T_t}\left(2G\sqrt{\left[\pi T_{coil}\left(\frac{2E-R_1\sqrt{2}}{2}\right)\right]^2 + H^2} + R_1^2\left[4-\pi-\pi\left(1-\sqrt{2}\right)^2\right]\right)\right](h_1 - h_{3,Isentropic})}$$

Isentropic Efficiency approaches 100% as frequency approaches infinity. Maximize rotational frequency first!

Example

Target Power (offshore oil platform)



Fluid

Process Fluid	methane	
Molecular Weight	16.04	Kg/Kmol
Specific Gas Constant	518.3	kJ/kg°K

STATE 1

Pressure	1100	psi
	7.58	Mpa
Temperature	100	°F
	310.9	°K

STATE 3

State 3 Pressure	100	psi
	0.69	Mpa

Expander

Target Power $\dot{W}_{PRE}$	30	kw	
Expander Frequency	60	htz	Limited by the drive assembly
R1	0.125	in.	Limited by manufacturing processes
Gap	0.005	in.	Limited by manufacturing processes

Constants given by the application

E_{system}

E (in.) \ H (in.)	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
0.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1.5	-	-	-	-	-	-	-	-	-	-	-	13.4%	13.5%	13.6%	13.7%	13.7%	13.8%	13.8%	13.8%	13.7%
2.0	-	-	-	-	-	16.5%	17.5%	18.1%	18.5%	18.7%	18.9%	18.9%	19.0%	18.9%	18.8%	18.8%	18.6%	18.5%	18.4%	18.2%
2.5	-	-	-	17.3%	19.2%	20.4%	21.1%	21.6%	21.8%	21.9%	21.9%	21.8%	21.7%	21.5%	21.3%	21.0%	20.8%	20.5%	20.3%	20.0%
3.0	-	-	17.2%	19.8%	21.4%	22.4%	23.0%	23.2%	23.2%	23.1%	23.0%	22.7%	22.4%	22.0%	21.7%	21.3%	21.0%	20.6%	20.2%	19.9%
3.5	-	15.0%	19.0%	21.3%	22.6%	23.3%	23.5%	**23.5%**	23.4%	23.1%	22.7%	22.3%	21.8%	21.4%	20.9%	20.5%	20.0%	19.6%	19.1%	18.7%
4.0	-	16.6%	20.2%	22.1%	23.0%	23.3%	23.3%	23.1%	22.7%	22.2%	21.7%	21.2%	20.6%	20.1%	19.6%	19.0%	18.5%	18.1%	17.6%	17.1%
4.5	-	17.7%	20.8%	22.3%	22.8%	22.8%	22.6%	22.1%	21.6%	21.0%	20.4%	19.7%	19.1%	18.5%	18.0%	17.4%	16.9%	16.4%	15.9%	15.5%
5.0	12.2%	18.5%	21.1%	22.1%	22.3%	22.0%	21.5%	20.9%	20.2%	19.5%	18.8%	18.2%	17.5%	16.9%	16.3%	15.8%	15.3%	14.8%	14.3%	13.8%
5.5	13.2%	19.0%	21.1%	21.7%	21.5%	21.0%	20.3%	19.6%	18.8%	18.1%	17.3%	16.6%	16.0%	15.4%	14.8%	14.2%	13.7%	13.2%	12.8%	12.4%
6.0	14.0%	19.2%	20.9%	21.1%	20.6%	19.9%	19.1%	18.2%	17.4%	16.6%	15.9%	15.2%	14.5%	13.9%	13.3%	12.8%	12.3%	11.9%	11.4%	11.0%
6.5	14.6%	19.4%	20.5%	20.3%	19.7%	18.8%	17.8%	16.9%	16.1%	15.3%	14.5%	13.8%	13.2%	12.6%	12.1%	11.5%	11.1%	10.7%	10.2%	9.9%
7.0	15.1%	19.3%	20.0%	19.5%	18.6%	17.6%	16.6%	15.7%	14.8%	14.0%	13.3%	12.6%	12.0%	11.4%	10.9%	10.4%	10.0%	9.6%	9.2%	8.9%
7.5	15.5%	19.1%	19.4%	18.7%	17.6%	16.5%	15.5%	14.5%	13.7%	12.9%	12.1%	11.5%	10.9%	10.4%	9.9%	9.4%	9.0%	8.6%	8.3%	8.0%
8.0	15.8%	18.9%	18.8%	17.8%	16.6%	15.5%	14.4%	13.5%	12.6%	11.8%	11.1%	10.5%	9.9%	9.4%	9.0%	8.6%	8.2%	7.8%	7.5%	7.2%
8.5	16.0%	18.5%	18.1%	16.9%	15.7%	14.5%	13.4%	12.5%	11.6%	10.9%	10.2%	9.6%	9.1%	8.6%	8.2%	7.8%	7.4%	7.1%	6.8%	6.5%
9.0	16.1%	18.2%	17.4%	16.1%	14.8%	13.6%	12.5%	11.6%	10.7%	10.0%	9.4%	8.8%	8.3%	7.9%	7.5%	7.1%	6.8%	6.5%	6.2%	5.9%
9.5	16.2%	17.7%	16.7%	15.3%	13.9%	12.7%	11.6%	10.7%	9.9%	9.3%	8.7%	8.1%	7.6%	7.2%	6.8%	6.5%	6.2%	5.9%	5.6%	5.4%
10.0	16.2%	17.3%	16.0%	14.5%	13.1%	11.9%	10.9%	10.0%	9.2%	8.6%	8.0%	7.5%	7.0%	6.6%	6.3%	6.0%	5.7%	5.4%	5.2%	5.0%

T_3 (F)

E (in.) \ H (in.)	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
0.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1.5	-	-	-	-	-	-	-	-	-	-	-	21.2	20.9	20.7	20.5	20.4	20.4	20.4	20.4	20.4
2.0	-	-	-	-	-	14.6	12.6	11.3	10.5	9.9	9.6	9.5	9.5	9.6	9.7	9.9	10.1	10.4	10.7	11.0
2.5	-	-	-	13.0	9.1	6.5	4.9	3.9	3.4	3.2	3.2	3.4	3.8	4.1	4.6	5.1	5.6	6.2	6.7	7.3
3.0	-	-	13.3	7.7	4.2	2.2	1.0	0.5	0.4	0.6	1.0	1.6	2.2	2.9	3.7	4.5	5.2	6.0	6.8	7.6
3.5	-	17.8	9.4	4.5	1.8	0.4	-0.2	-0.2	0.2	0.8	1.6	2.4	3.4	4.3	5.3	6.3	7.2	8.2	9.1	10.0
4.0	-	14.5	6.9	2.9	1.0	0.3	0.3	0.8	1.6	2.6	3.6	4.8	5.9	7.1	8.2	9.3	10.3	11.4	12.4	13.3
4.5	-	12.2	5.5	2.5	1.3	1.3	1.9	2.8	4.0	5.2	6.5	7.8	9.1	10.4	11.6	12.7	13.8	14.9	15.9	16.8
5.0	23.6	10.5	4.9	2.8	2.4	3.0	4.0	5.4	6.8	8.3	9.7	11.1	12.5	13.8	15.0	16.2	17.3	18.3	19.3	20.2
5.5	21.6	9.5	4.9	3.7	4.0	5.1	6.6	8.1	9.8	11.4	12.9	14.4	15.8	17.1	18.3	19.4	20.5	21.5	22.5	23.4
6.0	20.0	8.9	5.4	5.0	5.9	7.4	9.2	11.0	12.7	14.4	16.0	17.5	18.8	20.1	21.3	22.4	23.4	24.4	25.3	26.1
6.5	18.7	8.6	6.2	6.5	8.0	9.9	11.8	13.7	15.6	17.3	18.8	20.3	21.6	22.9	24.0	25.1	26.0	27.0	27.8	28.6
7.0	17.6	8.7	7.2	8.3	10.1	12.3	14.4	16.4	18.2	19.9	21.4	22.9	24.2	25.3	26.4	27.4	28.3	29.2	30.0	30.7
7.5	16.8	9.1	8.5	10.1	12.3	14.6	16.8	18.8	20.6	22.3	23.8	25.2	26.4	27.5	28.6	29.5	30.4	31.2	31.9	32.6
8.0	16.2	9.6	9.8	11.9	14.4	16.8	19.0	21.1	22.9	24.5	26.0	27.3	28.4	29.5	30.5	31.3	32.1	32.9	33.6	34.2
8.5	15.7	10.3	11.3	13.7	16.4	18.9	21.1	23.1	24.9	26.5	27.9	29.1	30.2	31.2	32.1	33.0	33.7	34.4	35.0	35.6
9.0	15.5	11.2	12.7	15.5	18.3	20.8	23.1	25.0	26.8	28.3	29.6	30.8	31.8	32.8	33.6	34.4	35.1	35.7	36.3	36.8
9.5	15.3	12.1	14.2	17.2	20.1	22.7	24.9	26.8	28.4	29.9	31.1	32.3	33.2	34.1	34.9	35.6	36.3	36.9	37.4	37.9
10.0	15.3	13.1	15.7	18.9	21.8	24.3	26.5	28.4	29.9	31.3	32.5	33.6	34.5	35.3	36.1	36.8	37.4	37.9	38.4	38.9

Thank You

QUESTIONS?

Insights

- Preliminary 1st order analysis at Helidyne showed a 1/1 rotor profile, E/H, is the most efficient. This research (2nd order analysis) shows that optimal rotor sizes are close to, but deviate from the 1/1 ratio due to the compressible affect of the fluid at the leak points (choked flow throat).

- First order and second order analysis agree on the qualitative behavior of efficiency when rotor size varies. Machine radius “E” is more dominant than rotor length “H” when determining isentropic efficiency. This is due to the natural shape and contours of the rotors.

- Calculation approach for the isentropic efficiency equation differs depending on the flow application. In the situation where mass flowrate is a constant and maximum power output is desired, a second iterative calculation is done to solve for the cavity pressure (cavity and throat states are coupled). If a target power is called for, state 2 pressure can be directly solved for and only the throat properties are iteratively calculated.

- If rotor dimensions are actively optimized, minimal losses occur with reduced rotational speed. For example, an expander optimized for 8000 RPM is compared with an expander optimized for 1800 RPM, the model shows an approximate 3% difference in isentropic efficiency. This allows easier design for auxiliary components.